Exact Algorithms and Heuristics for the Perfect Awareness Problem*

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Abstract. In this work, we describe an ongoing computational study on the Perfect Awareness Problem (PAP), which involves the spreading of information on social networks. The aim is to select an initial set of information-spreaders (seeds) so that, at the end of a propagation process, all members of that network are aware of the information. We have been developing heuristics based on the meta-heuristic Greedy Randomized Adaptive Procedure (GRASP) and, for an exact approach, we propose two Integer Programming (IP) formulations for the PAP.

Keywords: Information dissemination · Spreading of influence · Meta-heuristic · Integer Programming.

1 Introduction

Problem definition Consider a network modeled as a simple graph $G = (V, E)$, where an edge $\{u, v\} \in E$ indicates that the two individuals represented by vertices $u, v \in V$ are able to communicate with each other. We regard the propagation of any information as discretized in rounds. In each round, a vertex is in one of three states: *spreader*, *aware* or *unaware*. In the first case, the vertex is able to propagate the information, in the second, it has been given knowledge of it but cannot (yet) communicate it to its neighbors and, in the third, it has not even been informed by any of its neighbors. Initially, every vertex is unaware, except for those in a *seed set* $S \subseteq V$ wherein all vertices are spreaders. If a vertex v has at least one spreader neighbor in round i, then (the state of) v becomes aware in round $i + 1$. An aware vertex never reverts to unaware. Given a *threshold* function $t : V \to \mathbb{N}$, whenever a vertex $v \in V$ has at least $t(v)$ spreaders in its neighborhood in round i, v becomes a spreader in round $i + 1$. Clearly, every spreader can be regarded as aware even though we consider one state at a time. A spreader vertex's state never changes in future rounds.

Finally, with these notations in mind, the Perfect Awareness Problem (PAP) is formulated as: given G and t, the objective is to select a seed set of minimum size such that, at the end of the propagation process, no vertex remains unaware.

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Brief Literature review The PAP was proposed and proven to be $\mathbb{NP}$-hard in [2], where other complexity-related theoretical results, extended from known facts on the Target Set Selection and Dominating Set problems [1, 3], are also shown. Additionally, in [2], a heuristic for PAP is proposed, as well.

2 Our contributions

GRASP Recall that a generic heuristic based on GRASP [4] has two phases. Firstly, a feasible solution is incrementally constructed from greedy randomized criteria. Secondly, a local search is applied around this solution.

In this work, we propose four GRASP-based heuristics, which differ on the mechanism applied in the first phase. We also consider three different ways to calculate the benefits of inserting an element into a partial solution. Results from our experiments with these three methods show that two of them lead to better solutions than the heuristic proposed in [2] for 12 of the 17 instances presented therein.

Preprocessing In order to reduce instance size, we start by decomposing the original graph into connected components. Next, we collapse neighboring vertices with threshold 1. Lastly, we remove a vertex v whenever $t(v) = 1$ and, for each pair of neighbors $\{x, y\}$ of v, we add an edge $\{x, y\}$, if it does not exist. These three preprocessing steps allow us to solve smaller instances and combine their solutions into a feasible one for the original input.

IPs Besides, we propose two IP formulations for the PAP. The first one contains only a set of binary variables associated to vertices in each round. The second is more sophisticated and does not explicitly consider rounds. Instead, we work with three sets of binary variables. Two of them are associated to vertices and the last one is associated to pairs of arcs that replace the original edges.

3 Future developments

Ongoing investigations include new greedy criteria, refining the local search, as well as parameter adjustments in the GRASP heuristic. Regarding the IP models, we are developing *branch-and-cut* algorithms and probing new sets of constraints.

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