Chromatic number and oriented trees

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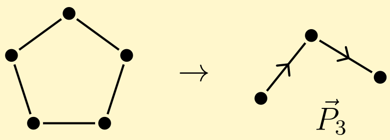
$G \to T$:= every orientation of G contains T



Any graph G contains all trees of order $\chi(G)$ (exercise!)

What if we consider orientations?

Theorem	$\chi(G) \geqslant t \implies G \to \vec{P}_t$	Gallai '66; Hasse '64; Roy '67; Vitaver '62	True for directed paths… (but **not** $\forall$ orientations!)
	$\mathcal{S}(t) := \{ S : S \text{ oriented star of order } t \}$		*maybe true* $\forall$ orientations if $n \geqslant 8$
Theorem	$\chi(G) \geqslant 2t-2 \implies G \to S \quad \forall\, S \in \mathcal{S}(t)$	Burr '80	For stars, a **factor** 2 appears.
	$\mathcal{T}(t) := \{ T : T \text{ oriented tree of order } t \}$		
Conjecture	$\chi(G) \geqslant 2t-2 \implies G \to T \quad \forall\, T \in \mathcal{T}(t)$	Burr '80	Is **that** true always?
Theorem	$\chi(G) \geqslant (t-1)^2 \implies G \to T \quad \forall\, T \in \mathcal{T}(t)$	Burr '80	
	$q(G) := \max\{ t : G \to T, \quad \forall\, T \in \mathcal{T}(t) \}$		Some graphs may contain larger trees…
Theorem	$\chi(G) \geqslant \binom{t}{2} \implies G \to T \quad \forall\, T \in \mathcal{T}(t)$	Addario-Berry, Sales, Havet, Reed, Thomassé '13	$q(G) \geqslant \dfrac{1+\sqrt{1+8\chi(G)}}{2}$
Theorem	$\chi(G) \geqslant t \cdot \log_2\lvert V(G)\rvert \implies G \to T \quad \forall\, T \in \mathcal{T}(t)$	Naia '18	$q(G) \geqslant \dfrac{\chi(G)}{\log_2\lvert V(G)\rvert}$
Theorem	$\forall\, p \in (0,1) \implies \chi\big(G(n,p)\big) \approx \dfrac{n}{\log_2 n}$ a.a.s.	Bollobás '88	Graphs typically have χ within a log factor of their order.
Corollary	$\forall\, p \in (0,1),\ \exists\, \varepsilon > 0 \implies G(n,p) \to T$ a.a.s. $\forall\, T \in \mathcal{T}\left(\varepsilon \dfrac{\chi(G(n,p))}{\log_2 n}\right)$	Naia '18	A **log-factor** usually, i.e., $q(G) > \dfrac{\chi(G)}{\log_2 \chi(G)}$ for almost every graph.

References . . . .

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