W[1]-Hardness of the k-Center Problem Parameterized by the Skeleton Dimension

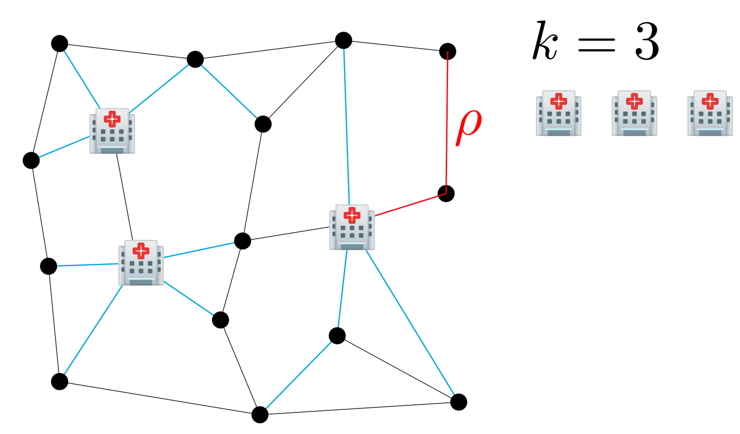
Johannes Blum

Universität Konstanz

The k-Center Problem

Given: Edge-weighted graph $G = (V, E)$
Integer k

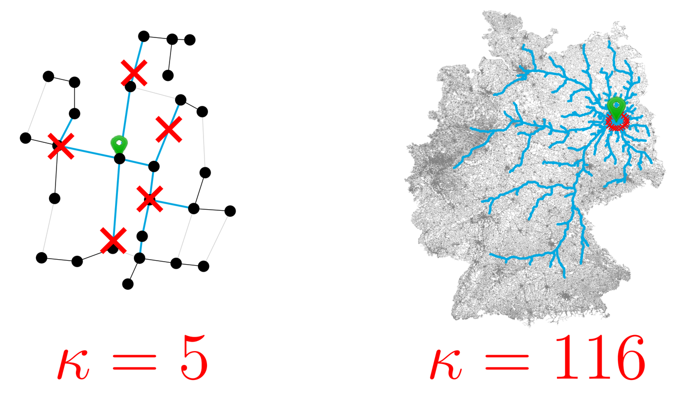
Goal: Set $C \subseteq V$ of k center vertices s.t. the maximum distance ρ from every $v \in V$ to the closest center is minimized



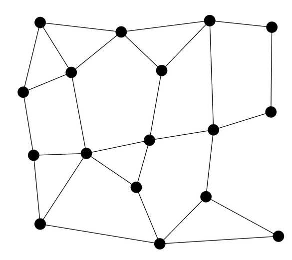
Models for Transportation Networks

Low Skeleton Dimension

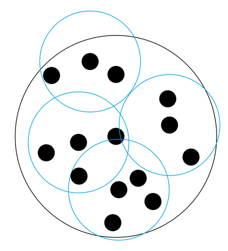
- ▷ Shortest path tree T_s
- ▷ Prune last third of every branch ⇒ skeleton T_s^*
- ▷ For every $r \geq 0$, skeleton T_s^* contains at most κ distinct vertices at distance r from s



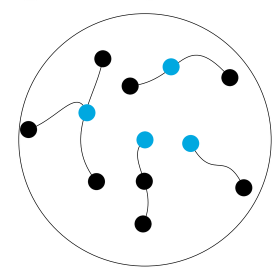
Planar Graphs



Low Doubling Dimension



Low Highway Dimension



Previous Work

NP-hard

2-approximation algorithm [Hochbaum & Shmoys '86]

No $(2 - \epsilon)$-approximation algorithm unless P=NP for

- ▷ planar graphs [Plesnk '80]
- ▷ graphs of constant doubling dimension [Feder & Greene '88]
- ▷ graph of highway dimension $\in \mathcal{O}(\log^2 |V|)$ [Feldmann '15]
- ▷ graph of skeleton dimension $\in \mathcal{O}(\log^2 |V|)$ [B. '19]

No exact $f(k) \cdot n^c$-time algorithm unless W[2]=FPT

Our Result

Extends [Feldmann & Marx '20]

On planar graphs of constant doubling dimension no exact $f(k, \kappa, h, p) \cdot n^c$-time algorithm, where

- k is the number of centers,
- κ is the skeleton dimension,
- h is the highway dimension, and
- p is the pathwidth,

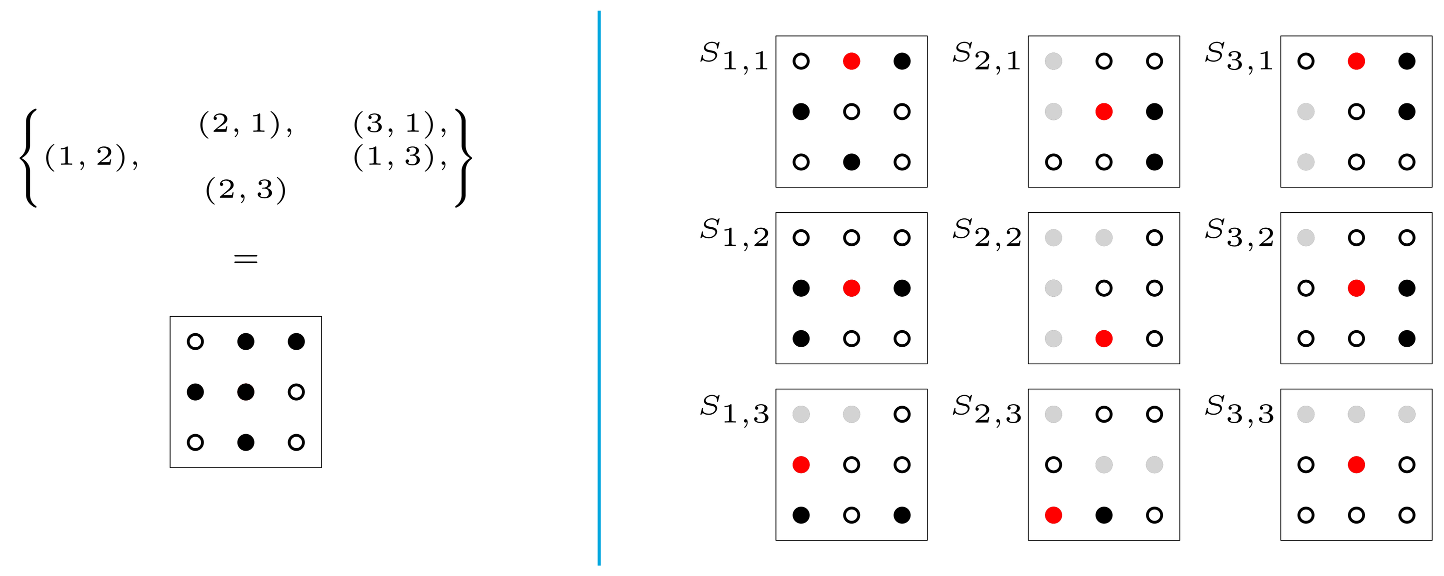
unless W[1]=FPT.

Reduction from the GridTiling$_\leq$ Problem

Given: χ^2 sets $S_{i,j} \subseteq [n]^2$ of pairs of integers

Goal: Select one pair $s_{i,j}$ from every $S_{i,j}$, such that

- if $s_{i,j} = (a, b)$ and $s_{i+1,j} = (a', b')$ we have $a \leq a'$, and
- if $s_{i,j} = (a, b)$ and $s_{i,j+1} = (a', b')$ we have $b \leq b'$.

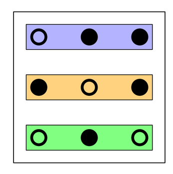


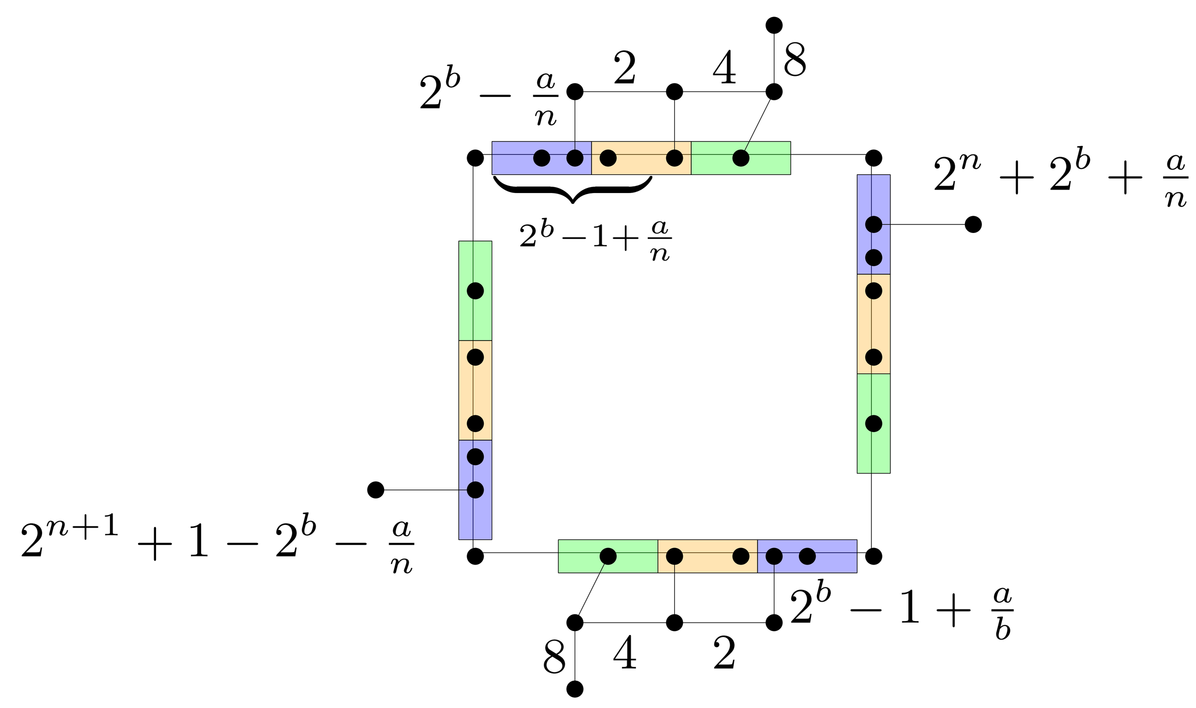
No exact $f(\chi) \cdot n^c$ time algorithm, unless W[1] = FPT.

[Marx & Sidiropoulos '14]

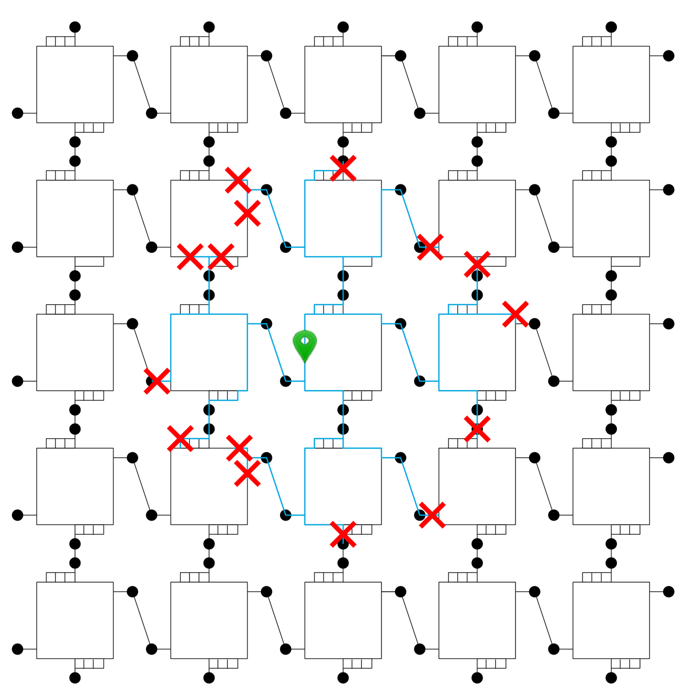
The Gadget $G_{i,j}$

$$S_{i,j} = \left\{ (1,2), \; (2,1), \; (2,3), \; (3,1), \; (1,3) \right\} =$$





The Whole Graph



- ▷ planar
- ▷ skeleton dimension $\kappa \in O(\chi)$
- ▷ constant doubling dimension
- ▷ highway dimension $h \in O(\chi^2)$
- ▷ pathwidth $p \in O(\chi)$

+

GridTiling$_\leq$ has no exact $f(\chi) \cdot n^c$ time algorithm, unless W[1] = FPT.

Our Result

Full Paper
https://arxiv.org/abs/2008.07252

Video
https://cloud.uni-konstanz.de/index.php/s/x6aBmTsQMg55gQQ